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ONE-DIMENSIONAL STATISTICAL MODEL OF ACTIVE TRANSFER OF SUBSTANCES IN MEMBRANES

A. V. Melkikh and V. D. Seleznev

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A matrix of kinetic coefficients that describe the processes of particle and heat transfer in a double membrane is obtained. The kinetic coefficients are considered as functions of the membrane parameters. Conditions are found under which the energy expended on transfer of one atom through the membrane is minimum. The obtained theoretical value of this energy is compared with the corresponding value for active transport of ions in biomembranes.

Membrane methods of separation and purification of substances find increasingly widespread application owing to their simplicity, economy, and low energy and material requirements [1]. Further development of this field of technology is hampered in many respects because of insufficient insight into the mechanisms of transfer of substances in membranes. Data on the mechanisms and technological application of the effects associated with heat fluxes are particularly incomplete. Investigation of the way membranes of living organisms function gives an idea of the great potentialities of energy flows [2]. Biological membranes, owing to liberation of the energy of metabolic processes, may act as pumps that create concentration drops for some ions. However, practical application of such effects is hindered by a lack of theoretical models of active transfer that take direct account of the energy sources and heat fluxes.

The present work proposes a simple theoretical model of the transfer of molecules or ions in a monolithic double solid membrane in the presence of a stationary heat source in the plane of contact of the layers. The model enables us to understand some features of active transfer of substances through membranes and to find the energy of the source required for transferring one particle.

We consider an infinite double membrane of thickness L that separates mixtures of gases or liquids (for example, solutions) with temperatures T_1 and T_2 and numerical densities of components n_{i1} and n_{i2}, respectively (Fig. 1). We place a stationary heat source with a surface power density W in the plane $x = L_1 < L$.

We will assume that the concentration of all the components of the liquid (gas) in the membrane material is so small that molecules or ions do not affect one another when transferred through it. This enables us to find the velocities of different components independently. Therefore, in the subsequent discussion we will consider the transfer of one component and will omit the subscript i.

For simplicity we will assume that the potential energy of the interaction of the membrane with the particles, averaged over thermal fluctuations, can be represented as shown in Fig. 2.

The energy U is reckoned from the level of the energy of the interaction of a particle with the solution. We ignore adsorption effects on the membrane surfaces.

We calculate particle and energy transfer between two neighboring potential wells on the basis of the assumption that the time spent by a particle in a well is much greater than the time taken by it to reach equilibrium with membrane molecules after the next jump. In this case for the densities of the particle and heat fluxes through an arbitrary barrier with the coordinate x we can write

$$I(x) = \frac{1}{4} n(x) v_t(x) \exp\left(-\frac{E(x)}{kT(x)}\right) \left\{-\frac{\Delta n_\text{b}}{n(x)} - \frac{\Delta T_\text{b}}{T(x)} \left(\frac{1}{2} + \frac{E(x)}{kT(x)}\right)\right\}, \tag{1}$$

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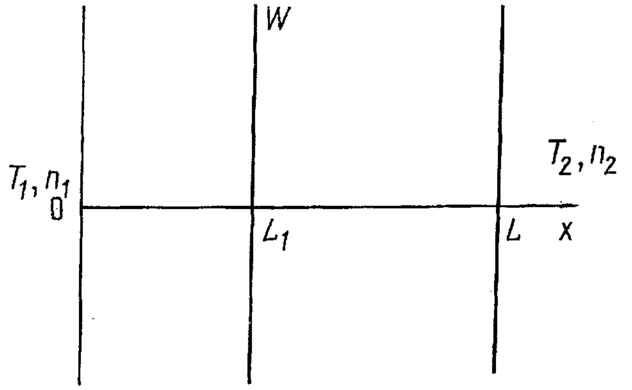

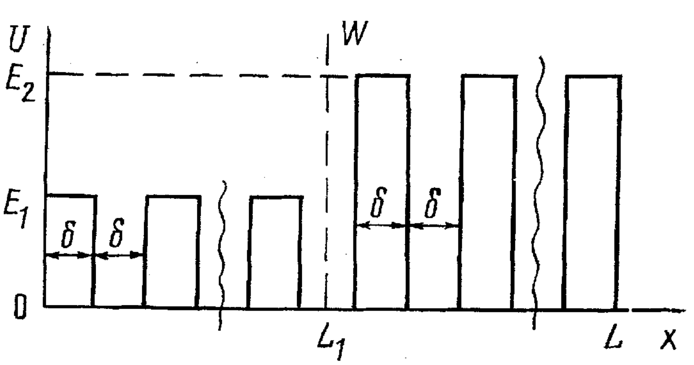

Fig. 1. Double membrane with a heat source.

Fig. 2. Profile of the particle potential energy in a double membrane.

$$q(x) = \frac{1}{4} n(x) v_t(x) \exp\left(-\frac{E(x)}{kT(x)}\right) \left\{ -\frac{\Delta n_b}{n(x)} \left(\frac{kT(x)}{2} + E(x)\right) - \right.$$

$$\left. - \frac{\Delta T_b}{T(x)} \left[\frac{9}{4} kT(x) + E(x) \left(1 + \frac{E(x)}{kT(x)}\right)\right]\right\} - \lambda \frac{\Delta T_b}{\delta}. \qquad (2)$$

where Δn_b and ΔT_b are the difference in the densities and temperatures in neighboring potential wells near the barrier in question; λ is the thermal conductivity coefficient of the membrane material; δ is the lattice spacing; v_t is the thermal velocity of particles of the transferred component; $E(x) = E_1$ for $0 < x < L_1$, $E(x) = E_2$ for $L_1 < x < L$.

Equations for $I(x)$ and $q(x)$ are introduced using the Maxwell distribution function for the particles in each well, and the densities of the counterflows (from the left well to the right one and vice versa) are calculated from the following formulas:

$$I_{l\to r} = \frac{1}{4} n_l v_{tl} \exp\left(-\frac{E}{kT_l}\right), \quad I_{r\to l} = \frac{1}{4} n_r v_{tr} \exp\left(-\frac{E}{kT_r}\right), (3)$$

$$q_{l\to r} = \frac{1}{4} n_l v_{tl} \exp\left(-\frac{E}{kT_l}\right)\left(\frac{kT_l}{2} + E\right), \qquad (3)$$

$$q_{r\to l} = \frac{1}{4} n_r v_{tr} \exp\left(-\frac{E}{kT_r}\right)\left(\frac{kT_r}{2} + E\right).$$

In going from (3) to (1) and (2) we assume that the temperature drop on the lattice spacing is small: $\Delta T_b/T \ll 1$.

We now assume that the differences in the particle densities and temperatures on both sides of the membrane are small, i.e.,

$$\frac{\Delta T}{\overline{T}} = \frac{T_1 - T_2}{\overline{T}} \ll 1, \quad \frac{\Delta n}{\overline{n}} = \frac{n_1 - n_2}{\overline{n}} \ll 1, \quad \overline{T} = \frac{T_1 + T_2}{2}, \quad \overline{n} = \frac{n_1 + n_2}{2},$$

and the heat source is limited to such an extent that temperature drops inside the membrane comparable with the average temperature $\overline{T}$ cannot be created. In this case we should expect that the distribution of the density n and temperature T over the membrane thickness will be piecewise linear, as shown in Fig. 3.

The linearity of the distributions $T(x)$ and $n(x)$ in homogeneous parts of the membrane follows from the fact that in the stationary case the heat and particle fluxes are the same in every cross section of each part of the membrane.

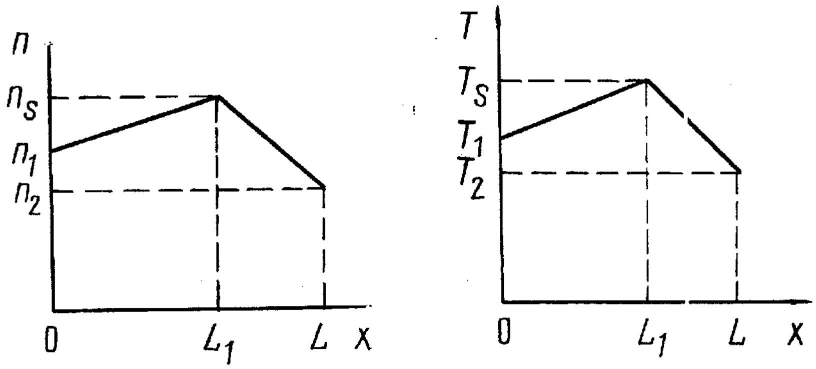

Fig. 3. Particle concentration and membrane temperature vs coordinate.

The drops in density Δn_b and temperature ΔT_b in neighboring potential wells will be the same within the homogeneous parts of the membrane and they change abruptly in going through the plane of contact:

$$\frac{\Delta n_{b1}}{\delta} = -\frac{n_s - n_1}{L_1}, \quad \frac{\Delta n_{b2}}{\delta} = -\frac{n_2 - n_s}{L - L_1}, \quad \frac{\Delta T_{b1}}{\delta} = -\frac{T_s - T_1}{L_1}, \quad \frac{\Delta T_{b2}}{\delta} = -\frac{T_2 - T_s}{L - L_1}.$$

With a prescribed heat source power density W we can find the unknown values of the density n_s and temperature T_s from the balance equations for the number of particles and the energy in the potential well with the coordinate $x = L_1$:

$$\kappa_1 \left\{ \varphi + \left(\frac{1}{2} + \frac{E_1}{kT} \right) \chi \right\} = \kappa_2 \left\{ \frac{\Delta n}{n} - \varphi + \left(\frac{\Delta T}{T} - \chi \right) \left(\frac{1}{2} + \frac{E_2}{kT} \right) \right\},$$

$$\kappa_1 \left\{ \varphi \left(\frac{k\bar{T}}{2} + E_1 \right) + \chi \left[\frac{9}{4} k\bar{T} + E_1 \left(1 + \frac{E_1}{kT} \right) \right] \right\} + W + \lambda_1 \chi \frac{\bar{T}}{L_1} + \lambda_2 \frac{\chi \bar{T} - \Delta T}{L - L_1} =$$

$$= \kappa_2 \left\{ \left(\frac{\Delta n}{\bar{n}} - \varphi \right) \left(\frac{k\bar{T}}{2} + E_2 \right) + \left(\frac{\Delta T}{\bar{T}} - \chi \right) \left[\frac{9}{4} k\bar{T} + E_2 \left(1 + \frac{E_2}{kT} \right) \right] \right\}, \tag{4}$$

$$\varphi = \frac{n_1 - n_2}{\bar{n}}; \quad \chi = \frac{T_1 - T_2}{\bar{T}}; \quad \kappa_i = \frac{1}{2} \bar{n} v_t \exp\left(-\frac{E_1}{k\bar{T}} \right) \frac{1}{l_i}; \quad l_i = \frac{L_i}{\delta}.$$

The system of algebraic equations (4) enables us to find φ and χ and, consequently, Δn_{b1}, Δn_{b2}, ΔT_{b1}, and ΔT_{b2}, and after substituting the latter into (1) and (2) we can easily obtain the following expressions for the fluxes of heat and the number of particles:

$$\begin{pmatrix} I \\ q_s \\ W \end{pmatrix} = L_{ik} \begin{pmatrix} k \dfrac{\Delta n}{\bar{n}} \\ \dfrac{\Delta T}{\bar{T}^2} \\ \dfrac{T_s - \bar{T}}{\bar{T}^2} \end{pmatrix},$$

where

$$q_s = \frac{q_1 + q_2}{2}; \quad L_{ik} = \begin{pmatrix} L_{nn} & L_{nq} & L_{nw} \\ L_{qn} & L_{qq} & L_{qq} \\ L_{wn} & L_{wq} & L_{ww} \end{pmatrix}$$

is the matrix of kinetic coefficients, similar to that described in [3].

We express the kinetic coefficients in terms of the parameters of the model as follows:

$$L_{nn} = \frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2} \frac{1}{k}, \tag{5}$$

$$L_{nq} = L_{qn} = \frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2} T \left(\frac{1}{2} + \frac{E_1 + E_2}{2kT} \right), \tag{6}$$

$$L_{nw} = L_{wn} = \frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2} T \frac{E_2 - E_1}{kT}, \tag{7}$$

$$L_{qq} = \left(\frac{\lambda_1}{L_1} + \frac{\lambda_2}{L - L_1} \right) \frac{T}{4} + \frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2} T \left\{ \frac{5}{4} kT + \frac{kT}{2} \left(\frac{\kappa_2}{\kappa_1} + \frac{\kappa_1}{\kappa_2} \right) + \frac{E_1 + E_2}{2} \left(1 + \frac{E_1 + E_2}{2kT} \right) \right\}, \tag{8}$$

$$L_{wq} = L_{qw} = \frac{T}{2} \left(\frac{\lambda_2}{L - L_1} - \frac{\lambda_1}{L_1} \right) + \frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2} T \left\{ (E_2 - E_1) \left(\frac{1}{2} + \frac{E_1 + E_2}{2kT} \right) + kT \left(\frac{\kappa_2}{\kappa_1} - \frac{\kappa_1}{\kappa_2} \right) \right\}, \tag{9}$$

$$L_{ww} = \left(\frac{\lambda_1}{L_1} + \frac{\lambda_2}{L - L_1} \right) T + \frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2} T \left\{ 2kT \left(\frac{\kappa_2}{\kappa_1} + \frac{\kappa_1}{\kappa_2} \right) + 4kT + \frac{(E_2 - E_1)^2}{kT} \right\}. \tag{10}$$

The diagonal kinetic coefficients L_{nn} and L_{qq} describe the permeability of the double membrane to particles (L_{nn}) and heat (L_{qq}) due to thermodynamic forces conjugate to these fluxes, and the coefficient L_{ww} is associated with the rate of source heat removal on both sides of the membrane due to the temperature drop $(T_s - \overline{T})/T^2$.

The cross coefficients L_{nq} and L_{qn} correspond to the well-known phenomena of thermal creep and mechanocaloric heat transfer [4] for double membranes. In the case $W = 0$ a thermomolecular difference in the densities of the component in question that can be found from the condition $I = 0$ will be created in the closed system of two volumes joined by a membrane and having a maintained temperature difference. Based on this condition we can easily obtain from expressions (5) and (6) that

$$\frac{\Delta n}{n} = - \frac{L_{nq}}{L_{nn}} \frac{\Delta T}{k\overline{T}^2} = - \left(\frac{1}{2} + \frac{E_1 + E_2}{2kT} \right) \frac{\Delta T}{T}. \tag{11}$$

Taking into account that for an ideal gas

$$\frac{\Delta n}{n} = \frac{\Delta p}{p} - \frac{\Delta T}{T}, \tag{12}$$

we can calculate from expression (11) the index of the thermomolecular pressure difference (t.p.d.) for rarefied gases:

$$\gamma = \frac{\Delta p / p}{\Delta T / T} = \frac{1}{2} - \frac{E_1 + E_2}{2kT}. \tag{13}$$

It is pertinent to note that in the case of a homogeneous membrane ($E_1 = E_2$) the t.p.d. index (13) coincides with the result of [5].

The cross effect described by the coefficient L_{nw} corresponds to the flux of the number of particles through an asymmetric double membrane due to the temperature drop $(T_s - \overline{T})/T^2$. In rarefied gas motion in a composite channel a similar effect (that of accommodation pumping) was found and investigated in the works of Hobson and Edmonds [6, 7]. The analog of this effect in conductors with an electric current can be called the Seebeck phenomenon [4]. A common feature of the above effects is the fact that opposing temperature gradients in different parts of heterogeneous membranes (conductors) induce particle fluxes (electric currents) that differ in magnitude and are opposite in direction. The resultant particle flux (electric current) manifests itself in the pressure (potential) difference created. The corresponding resultant fluxes become zero as the characteristics of the unlike parts of the membrane converge. Consequently, the cross coefficients that describe these phenomena should be proportional to the differences in the parameters that characterize the transfer of the component in question in the unlike parts of the system. The diffusion activation energy E_i is such a characteristic for the theoretical model proposed, and therefore $L_{nw} \sim (E_2 - E_1)$. The difference structure of the expressions for the kinetic coefficient in question predetermines a change in direction for the particle flux upon interchange of the homogeneous parts of the membrane.

Thus, by maintaining a nonuniform temperature field in an asymmetric membrane with a maximum (minimum) at the contact we can induce a pump effect whose magnitude can be easily found from the condition $I = 0$ at $\Delta T = 0$ in (5):

$$\frac{\Delta n}{n} = -\frac{L_{nw}}{L_{nn}} \frac{T_s - \overline{T}}{k\overline{T}^2} = \frac{E_1 - E_2}{kT} \frac{T_s - \overline{T}}{\overline{T}}. \tag{14}$$

As evident from (14), for $T_s > \overline{T}$ the particle density will be larger on the side of the membrane where the diffusion activation energy is larger. This is explained by the fact that the thermal creep directed from the contact is larger in magnitude for the part of the membrane that is characterized by the activation energy E_2.

According to (14) the relative density difference $\Delta n/n$ (the pump effect) can attain values of the order of unity; particularly large effects are to be expected at low temperatures ($(E_2 - E_1)/kT \gg 1$). It is pertinent to note that the effect of accommodation pumping [8] for $(T_s - \overline{T})/T \sim 1$ can hardly exceed the difference in the accommodation coefficients for tangential momentum $(\alpha_1 - \alpha_2)$ on different parts of a composite channel ($\Delta n/n < (\alpha_1 - \alpha_2) \lesssim 0.1$). Therefore, to ensure the maximum pump effect, we need to change from the channels with a large diameter of ~2 cm that were used in [8] to a set of parallel small-radius channels in which surface diffusion of gas atoms with the activation energies E_1 and E_2 will be the main mechanism for transferring them.

The kinetic coefficient L_{wn} (equal to L_{nw}) describes heat liberation (absorption) at the interface of the dissimilar parts of an asymmetric membrane due to the particle flux through the membrane resulting from the density drop $\Delta n/n$. This effect is analogous to the Peltier effect in conductors of dissimilar materials. The corresponding effect for rarefied gases in the component parts of a channel is described in [3]. The main reason for these phenomena is the difference in the mechanocaloric heat fluxes in the dissimilar parts of the asymmetric membrane (conductors). The relative temperature difference at the channel interface in [3] is no more than 10^{-3}, as a rule; the heat fluxes are also very small by virtue of the gas rarefaction, which narrows the field of possible application of the gas analog of the Peltier effect in channels. However, by going over to a set of small channels that provides a transfer regime in which surface diffusion will be the prevailing mechanism the magnitude of the effect can be increased significantly. By using explicit expressions for the kinetic coefficients L_{wn} and L_{nw} we can easily obtain that the heat liberation in the contact region per particle will be $E_2 - E_1$.

The kinetic coefficient L_{qw} characterizes the phenomenon of heat removal from the contact of the dissimilar parts of a composite membrane, which concerns the difference in the magnitudes of the heat fluxes $(|q_2| - |q_1|)$ that is induced by the temperature drop $(T_s - \overline{T})/\overline{T}^2$. The magnitudes $|q_1|$ and $|q_2|$ will differ only for asymmetric membranes, and therefore the coefficient L_{qw} should have a difference character, which is confirmed by the form of expression (9).

If a constant temperature difference is maintained at the ends of a composite membrane with a linear temperature distribution over the membrane thickness, the heat fluxes in the dissimilar parts of the membrane will coincide in direction but their magnitude will be different because of the difference in the transfer parameters. As a result, the temperature drop $\Delta T = T_1 - T_2$ will lead to heat liberation at the contact that is equal to $|q_2| - |q_1|$ and is characterized by the coefficient L_{wq}.

It is noteworthy that the matrix of kinetic coefficients L_{ik} obtained is symmetric, i.e., the Onsager reciprocal relation

$$L_{ik} = L_{ki} .$$

holds for these coefficients.

We now consider the effect of active particle transfer under the action of a steady heat source W. We will assume that $\Delta n/n = \Delta T/T = 0$, and therefore, according to (5), the temperature drop

$$\frac{T_s - \overline{T}}{\overline{T}^2} = \frac{W}{L_{ww}} . \tag{15}$$

will be established in the membrane.

This drop, according to (5), creates the flux of the number of particles (the pump effect)

$$I = L_{nw} \frac{T_s - \overline{T}}{\overline{T}^2} = \frac{L_{nw}}{L_{ww}} W . \tag{16}$$

Based on (7) we can easily find the energy ε required to transfer one particle through the membrane:

$$\varepsilon = \frac{W}{I} = \frac{L_{ww}}{L_{nw}} = \frac{\left(2\frac{\kappa_1}{\kappa_2} + 2\frac{\kappa_2}{\kappa_1} + 4\right) kT + \frac{(E_2 - E_1)}{kT} + \frac{\kappa_1 + \kappa_2}{\kappa_1 \kappa_2}\left(\frac{\lambda_1}{L_1} + \frac{\lambda_2}{L - L_1}\right)}{E_1 - E_2} . \tag{17}$$

To simplify, we denote $E_i/kT \equiv E_i$, $E_1 - E_2 = \Delta$, $x = L_1/(L - L_1)$, and then

$$\varepsilon = \frac{1}{\Delta}\left\{2e^{-\Delta} x^{-1} + 2e^{\Delta} x + 4 + \Delta^2 + \frac{\kappa_1 + \kappa_2}{\kappa_1 \kappa_2}\left(\frac{\lambda_1}{L_1} + \frac{\lambda_2}{L - L_1}\right)\right\} . \tag{18}$$

As is evident from (18), the value of ε is determined by the activation energy difference $\Delta = E_1 - E_2$, the coordinate of the location of the heat source $x = L_1/(L - L_1)$, and the thermal conductivities λ_1 and λ_2 of both parts of the membrane. It is clear that the most efficient operation of this pump corresponds to the minimum value of ε.

We find the conditions under which the expenditure of energy on pumping will be the smallest. For this purpose we find the minimum of the function $\varepsilon = \varepsilon(\Delta, X, \lambda_1, \lambda_2)$; ε depends on λ_i linearly, and λ_i is greater than 0. That is, to ensure the minimum of ε, we need to decrease λ_i to zero as far as possible ($\varepsilon \to \varepsilon_{\min}$ as $\lambda_i \to 0$). Thus, the heat flux over the membrane material is a parasitic process. To ensure a sufficiently high efficiency of pump operation, it is essential that the density of the flux of heat transferred by the particles of the penetrating component be at least a noticeable fraction of W.

With respect to the other variables $\Delta = E_1 - E_2$ and $X = (L_1/(L - L_1))$ the function $\varepsilon(\Delta, X)$ has a minimum determined from the conditions

$$\frac{\partial \varepsilon}{\partial \Delta} = 0 , \quad \frac{\partial \varepsilon}{\partial X} = 0 . \tag{19}$$

By taking the derivatives of (19) and setting them equal to zero we obtain

$$\varepsilon_{\min} = 2\sqrt{8}\, kT , \quad E_1 - E_2 = \sqrt{8}\, kT , \quad x = \exp(-\sqrt{8}) .$$

Thus, the minimum energy required to transfer one particle is $2\sqrt{8}\,kT$ provided that the thermal conductivity of the membrane material is negligibly small. The optimum difference in the activation energies equals $\sqrt{8}\,kT$, and the best location of the energy source is characterized by the parameter $L_1/(L - L_1) = \exp(-\sqrt{8})$.

As applied to biomembranes, we can expect that the most energy-efficient mechanisms of operation of active transport were selected in the process of biological evolution. Then biomembrane ion pumps should have near-optimum characteristics. As a comparison shows, the value of $\varepsilon_{\min}$ obtained is, indeed, in good agreement with experimental data for some membranes. For example, energy equal to $\sim 8kT$ is expended on transferring one Ca^{2+} ion, in the Ca^{2+}-pump of sarcoplasmatic reticulum cells [9]. We can assume that the difference between the experimental and theoretical values of energy transfer $\varepsilon_{\text{exp}} - \varepsilon_{\text{theor}} \sim 2.5kT$ arises because $\lambda_i \not\equiv 0$.

However, it is pertinent to note that there are biological pumps (for example, $Na^{+}-K^{+}$-adenosine triphosphatase) in which $\sim 3.2kT$ is expended on transferring one ion [9], i.e., less than the minimum estimate of ε obtained by us. This suggests that some biological membranes have a structure and associated mechanisms of active transfer (for example, gate ones [10]) that enable them to expend the energy of metabolic processes more economically. To describe these mechanisms, more detailed theoretical models are needed.

As a result of the steady action of active particle transfer when at least one of the volumes between which there is the membrane is limited, a constant density difference Δn^{st} is established over time. The value of Δn^{st} can be found from the condition of vanishing of the particle flux I due to the joint action of the mechanisms of active and passive transfer:

$$I = L_{nn}k\frac{\Delta n^{\text{st}}}{\bar{n}} + L_{nw}\frac{T_s - \bar{T}}{\bar{T}^2} = 0\,. \tag{20}$$

In view of (6) we can easily obtain from (20) that

$$\frac{\Delta n^{\text{st}}}{\bar{n}} = \frac{L_{nw}}{L_{nn}L_{ww}}\,W = \frac{E_1 - E_2}{kT}\;\frac{W}{2\dfrac{\kappa_1}{\kappa_2} + 2\dfrac{\kappa_2}{\kappa_1} + 4 + \left(\dfrac{E_2 - E_1}{kT}\right)^2}\;\frac{\kappa_1 + \kappa_2}{\kappa_1\kappa_2}\,. \tag{21}$$

For the optimum values of $\Delta = \sqrt{8}$ and $X = \exp(\sqrt{8})$ we find

$$\frac{\Delta n^{\text{st}}}{\bar{n}} = W\frac{2\sqrt{8}}{8 + 8}\,\frac{1}{\kappa_2} = \frac{W}{\kappa_2\sqrt{8}} = \frac{W}{\kappa_1\sqrt{8}}\,, \tag{22}$$

since $\kappa_1 = \kappa_2$ for optimum Δ and X.

We estimate Δn^{st} created by a biological pump. We take into account that the membrane thickness $L \sim 70$ Å, $v_t \sim 100$ m/sec, $\delta \sim 3$ Å. Furthermore, we estimate the heat source density W, knowing the ATP decay energy $Q \sim 16kT$ and the pump operating frequency $\nu \sim 2 \cdot 10^3\ \text{sec}^{-3}$ [11]. By substituting these values into Eq. (22) we obtain that the observed drop $\Delta n^{\text{st}} \sim 0.1$ mole/liter can be produced for $E_2 \sim 7kT$. At the same time we should take into account that the model given in the present work is correct only in the domain of linear thermodynamics (in the case of smallness of all forces) and cannot be applied in full measure to biological membranes since $\Delta n/n \sim 1$ in them. Thus, the proposed theoretical model of particle transfer in asymmetric membranes with different diffusion activation energies shows the fundamental importance of taking into account heat fluxes and energy liberation at the interface of the dissimilar parts of these membranes. The cross coefficients obtained describe transfer phenomena that are in qualitative agreement with those observed on membranes of cells of living organisms. The cross effects described in the present work can be used in engineering practice, for example, to produce pumping devices, separators, current sources, etc.

NOTATION

T_1, T_2, solution temperatures to the left and to the right of the membrane; n_{i1}, n_{i2}, numerical densities of particles of the i-th component to the left and to the right of the membrane; x, current coordinate in the membrane; L_1, coordinate of the heat source; L, membrane thickness; W, heat source power density; δ, distance between neighboring potential wells; E_1, E_2, diffusion activation energies in the left and right parts of the membrane; U, particle potential energy in the membrane; Δn_b, ΔT_b, difference in the gas densities and temperatures in neighboring potential wells; k, Boltzmann constant; λ_1, λ_2, thermal conductivity coefficients in both parts of the membrane material; v_t, thermal velocity of particles; I, particle flux density in the membrane; q, heat flux density in the membrane; T_s, n_s, particle temperature and density at the contact of the parts of the membrane; $\overline{T}$, $\overline{n}$, average temperature and density of particles in the membrane; q_s, heat flux at the contact of the membrane layers; L_{ik}, matrix of kinetic coefficients; p, gas pressure; γ, index of the thermomolecular pressure difference; α_1, α_2, accommodation coefficients of the particle tangential momentum in the dissimilar parts of a composite channel; ε, energy required to transfer one particle through the membrane.

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